# Intrinsic anomalous Hall effect in spin-polarized two-dimensional electron gases with Dresselhaus spin-orbit interaction

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The intrinsic anomalous Hall effect in spin-polarized two-dimensional electron gases with Dresselhaus spin-orbit interaction is studied within the Kubo-Streda formalism. We find that when the $k^3$ term of Dresselhaus interaction is taken into account, in the weak impurity scattering limit the intrinsic anomalous Hall conductivity is not zero and its absolute value increases with the increment of the Fermi energy and the thickness of the quantum well when both subbands are partially occupied. This result is opposite to the existing conclusion of Rashba spin-orbit interaction in which the anomalous Hall conductivity vanishes in the same situation.

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## I. INTRODUCTION

The theoretical discussion of anomalous Hall effect (AHE) in ferromagnetic metals and semiconductors has a long controversial history. Karplus and Luttinger[1,2] gave the first theoretical explanation of AHE and they presented an anomalous Hall coefficient proportional to the square of the ordinary resistivity. Later, two mechanisms of AHE based on the influence of disorder scattering in imperfect crystals, termed by skew scattering and side jump, were given by Smit[3] and Berger,[4] respectively. Interestingly, the intrinsic mechanism of AHE, resulting from the spin-orbit interaction (SOI) of conduction electrons which may induce a nonzero Berry phase or magnetic monopole in the momentum space, is renewed currently.[5,6] Moreover several experiments in ferromagnetic semiconductors can be quantitatively interpreted in terms of the intrinsic AHE mechanism.[5–10] Effects of disorder on the AHE in two-dimensional spin-polarized electron gases with Rashba SOI have been theoretically investigated by several groups.[11–20] Even for such a simple model of Rashba SOI, however, a series of previous studies yielded a multitude of results with discrepancies. Recently Nunner *et al.*[19] addressed this question on the origin of the previous discrepancies and found that all contributions to the anomalous Hall conductivity vanish to leading order in disorder strength when both of the spin-split bands are occupied. On the other hand, the intrinsic spin Hall effect (SHE) was also studied in the paramagnetic electronic systems with SOI.[21,22] The AHE and the SHE reflect the charge and spin aspects of electron transport, respectively, and have some common features as their physical origin stems from the same SOI of conduction electrons. Inoue *et al.*[15] also presented the strong similarity between AHE and SHE. Note that for pure Rashba SOI the spin Hall conductivity becomes zero even for a weak disorder scattering,[23–27] however, a nonzero spin Hall conductivity appears when the cubic terms of Dresselhaus SOI is included.[28] So, analogous to the SHE, what is the novel property of the AHE in two-dimensional spin-polarized electron gases with Dresselhaus SOI in contrast to the case for Rashba SOI? In this paper we will address this question and investigate a disordered two-dimensional electron gas with the intrinsic Dresselhaus-type SOI within the Kubo-Streda formalism to calculate the anomalous Hall conductivity. We will show that the intrinsic anomalous Hall conductivity in a Dresselhaus-type two-dimensional electron gas with uniform exchange splitting is nonvanishing even when both subbands are occupied, which is opposite to the existing conclusion of Rashba SOI. In fact the $k^3$ term of Dresselhaus SOI plays a key role in this nonzero conductivity, which is analogous to the intrinsic SHE. The numerical calculation is performed for a GaSb quantum well (QW) with Dresselhaus SOI in the weak impurity scattering limit and it is shown that when the $k^3$ term of Dresselhaus SOI is taken into account, the absolute value of the intrinsic anomalous Hall conductivity increases with the increment of the Fermi energy and the thickness of QW when both subbands are partially occupied.

## II. MODEL HAMILTONIAN

The Dresselhaus SOI arises from the asymmetry of the crystal itself[29] and in the bulk crystal is described by Hamiltonian[30,31]

$$\tilde{H}_{so} = \gamma[\sigma_x k_x(k_y^2 - k_z^2) + \text{c.p.}], \quad (1)$$

where $\gamma$ is the spin-orbit coefficient for the bulk semiconductor, $\sigma_\alpha$ ($\alpha = x,y,z$) are the Pauli matrices, $k_\alpha$ are the electron wave vector components, and c.p. stands for the cyclic permutation needed to obtain the remaining terms of the Hamiltonian. In a QW grown in the crystallographic direction [001] with thickness $a$ while $k_x$ and $k_y$ are good quantum numbers, the confinement along the $z$ axis is approximately realized by taking $\langle k_z\rangle = 0$ and $\langle k_z^2\rangle \simeq (\pi/a)^2$ for the lowest energy band, then the Dresselhaus SOI term [Eq. (1)] becomes[32,33]

$$H_{so} = \gamma[\sigma_x k_x(k_y^2 - \langle k_z^2\rangle) + \sigma_y k_y(\langle k_z^2\rangle - k_x^2)]. \quad (2)$$

Here, we take $k_x = k\cos\theta$ and $k_y = k\sin\theta$. The Dresselhaus SOI in Eq. (2) contains terms both linear and cubic in $k$.

We consider a spin-polarized two-dimensional electron gas with Dresselhaus SOI

 

$$H_0 = \frac{\hbar^2 k^2}{2m} + H_{so} - h_0\sigma_z, \tag{3}$$

where $m$ is the effective mass of the conduction electron and $h_0$ is the exchange field. Equation (3) can be written as a general form

$$H_0 = \varepsilon(k) + \sum_\alpha d_\alpha \sigma_\alpha \tag{4}$$

with $\varepsilon(k) = \hbar^2 k^2/(2m)$. Here, for the case described by Eq. (3) one has $d_x = -\gamma_0 k_x + \gamma k_x k_y^2$, $d_y = \gamma_0 k_y - \gamma k_y k_x^2$, and $d_z = -h_0$ with $\gamma_0 \equiv \gamma\langle k_z^2\rangle$. The eigenenergies of Hamiltonian (3) are

$$E_\pm(k,\theta) = \frac{\hbar^2 k^2}{2m} \pm \lambda(k,\theta), \tag{5}$$

where

$$\lambda(k,\theta) = \sqrt{\sum_\alpha d_\alpha^2} = \sqrt{h_0^2 + \gamma_0^2 k^2 + f_1(\theta)k^4 + f_2(\theta)k^6} \tag{6}$$

with $f_1(\theta) = -\gamma_0\gamma \sin^2 2\theta$ and $f_2(\theta) = \frac{1}{4}\gamma^3 \sin^2 2\theta$. The two spin-dependent dispersion branches $E_\pm(k,\theta)$ are angular anisotropic. For a realistic calculation we need to consider the scattering from impurities with a disorder potential $V(\mathbf{r})$. For simplicity, we consider $H_0$ in Eq. (3) with nonmagnetic impurities with short-ranged potential: $V(\mathbf{r}) = V_0 \Sigma_i \delta(\mathbf{r} - \mathbf{R}_i)$, where $V_0$ is the strength of the impurities. Thus, the Hamiltonian of our model is $H = H_0 + V(\mathbf{r})$. The retarded Green's function of the clean system is

$$G^{(0)R}(\mathbf{k},E) = \frac{E - \varepsilon(k) + \Sigma_\alpha d_\alpha \sigma_\alpha}{(E - E_+ + i0^+)(E - E_- + i0^+)}. \tag{7}$$

For a given Fermi energy $\epsilon_F$, the Fermi wave numbers $k_\pm$ of the two subbands are dependent on $\theta$ and one can get $k_\pm$ by means of numerical solving of the following equation:

$$\epsilon_F = \frac{\hbar^2 k_\pm^2}{2m} \pm \lambda_\pm \tag{8}$$

with $\lambda_\pm \equiv \lambda(k_\pm,\theta)$. Including the self-energy $\Sigma^R$, the retarded Green's function can be written as $G^R(\mathbf{k},E,\Sigma^R) = (E - H_0 - \Sigma^R)^{-1}$. We calculate the self-energy $\Sigma^R$ using the Born approximation[11,19]

$$\Sigma^R = n_i V_0^2 \int \frac{d^2k}{(2\pi)^2} G^{(0)R}(\mathbf{k},E) \tag{9}$$

$$= -i\left(\Gamma\sigma_0 + \sum_\alpha \Gamma_\alpha \sigma_\alpha\right) \tag{10}$$

with

$$\Gamma = \frac{1}{4} n_i V_0^2 \int \frac{d\theta}{2\pi}(\nu_+ + \nu_-), \tag{11}$$

$$\Gamma_\alpha = \frac{1}{4} n_i V_0^2 \int \frac{d\theta}{2\pi}\left(\frac{\nu_+}{\lambda_+} d_\alpha^+ - \frac{\nu_-}{\lambda_-} d_\alpha^-\right), \tag{12}$$

where $n_i$ is the density of the impurity, $d_\alpha^\pm = d_\alpha(k_\pm,\theta)$, and $\nu_\pm$ is related to the density of states at the Fermi levels of the two subbands

$$\nu_+ = \frac{m\Theta(\epsilon_F - h_0)}{\left|\hbar^2 + \frac{m}{\lambda_+}[\gamma_0^2 + 2f_1(\theta)k_+^2 + 3f_2(\theta)k_+^4]\right|}, \tag{13}$$

$$\nu_- = \frac{m}{\left|\hbar^2 - \frac{m}{\lambda_-}[\gamma_0^2 + 2f_1(\theta)k_-^2 + 3f_2(\theta)k_-^4]\right|}. \tag{14}$$

Here $\Theta(x)$ is the Heaviside step function.

Thus, the impurity averaged Green's function is given

$$G^R = \frac{[E - \varepsilon(k) + i\Gamma]\sigma_0 + \Sigma_\alpha(d_\alpha - i\Gamma_\alpha)\sigma_\alpha}{[E - \varepsilon(k) + i\Gamma]^2 - \Sigma_\alpha(d_\alpha - i\Gamma_\alpha)^2}. \tag{15}$$

In the limit of small $\Gamma_\alpha$, the retarded Green's function can also be written as[19]

$$G^R = G_0^R \sigma_0 + \sum_\alpha G_\alpha^R \sigma_\alpha, \tag{16}$$

where $G_0^R = \frac{1}{2}(G_+^R + G_-^R)$, $G_\alpha^R = \frac{1}{2} g_\alpha (G_+^R - G_-^R)$ with $G_\pm^R = (E - E_\pm + i\Gamma_\pm)^{-1}$, $\Gamma_\pm = \Gamma \pm \Sigma_\alpha d_\alpha \Gamma_\alpha/\lambda$, and $g_\alpha = (d_\alpha - i\Gamma_\alpha)\lambda/(\lambda^2 - i\Sigma_\alpha d_\alpha \Gamma_\alpha)$.

## III. GENERAL EXPRESSION FOR THE ANOMALOUS HALL CONDUCTIVITY

We use the Kubo-Streda formalism[34,35] to calculate the off-diagonal conductivity $\sigma_{yx}$, which can be decomposed into $\sigma_{yx} = \sigma_{yx}^{\mathrm{I}} + \sigma_{yx}^{\mathrm{II}}$.[11,19,36] Here $\sigma_{yx}^{\mathrm{I}}$ results from the contribution of electrons at the Fermi surface and $\sigma_{yx}^{\mathrm{II}}$ contains the contribution of all filled states below the Fermi energy, with

$$\sigma_{yx}^{\mathrm{I}} = \frac{e^2\hbar}{4\pi V}\mathrm{Tr}\langle v_y[G^R(\epsilon_F) - G^A(\epsilon_F)]v_x G^A(\epsilon_F) - v_y G^R(\epsilon_F) v_x [G^R(\epsilon_F) - G^A(\epsilon_F)]\rangle, \tag{17}$$

$$\sigma_{yx}^{\mathrm{II}} = \frac{e^2\hbar}{4\pi V}\int_{-\infty}^{\infty} dE f(E)\mathrm{Tr}\left\langle v_y G^R(E) v_x \frac{\partial G^R(E)}{\partial E} - v_y \frac{\partial G^R(E)}{\partial E} v_x G^R(E) - v_y G^A(E) v_x \frac{\partial G^A(E)}{\partial E} + v_y \frac{\partial G^A(E)}{\partial E} v_x G^A(E)\right\rangle, \tag{18}$$

where $\langle\cdots\rangle$ means the disorder average, the trace is taken over wave vectors and band index, $f(E)$ is the Dirac-Fermi distribution function, and the velocity operators are $v_\alpha = \partial H_0/(\hbar\,\partial k_\alpha)$. In the weak scattering limit we will omit the contributions of $G^R G^R$ and $G^A G^A$ in Eq. (17) since these terms are of higher order in the disorder scattering rate $\Gamma$.[19,20] Moreover, for $\sigma_{yx}^{\mathrm{II}}$ it is sufficient to calculate the bare bubble contribution.[19,36,37]

The bare contribution of $\sigma_{yx}^{\mathrm{II}}$ in the clean limit yields

$$\sigma_{yx}^{\mathrm{II}} = \frac{2e^2}{\hbar}\int \frac{d^2k}{(2\pi)^2}\left[\frac{\delta(\epsilon_F-E_+)+\delta(\epsilon_F-E_-)}{(E_+-E_-)} + \frac{f(E_+)-f(E_-)}{(E_+-E_-)^2}\right]\sum_{ijk}\epsilon_{ijk}\frac{\partial d_i}{\partial k_y}\frac{\partial d_j}{\partial k_x}\frac{d_k}{\lambda}$$
$$= \frac{e^2h_0}{2\hbar(2\pi)^2}\int d\theta\left[\frac{\nu_+\Lambda(k_+,\theta)}{\lambda_+^2}+\frac{\nu_-\Lambda(k_-,\theta)}{\lambda_-^2} + \Theta(\epsilon_F-h_0)\int_0^{k_+}kdk\frac{\Lambda(k,\theta)}{\lambda^3}-\int_0^{k_-}kdk\frac{\Lambda(k,\theta)}{\lambda^3}\right], \quad (19)$$

where $\Lambda(k,\theta)=-\gamma_0^2+\gamma_0\gamma k^2+\frac{3}{4}\gamma^2k^4\sin^2 2\theta$. If we neglect the $k^3$ term in Eq. (2), $\sigma_{yx}^{\mathrm{II}}$ can be analytically calculated and yields

$$\sigma_{yx}^{\mathrm{II}} = \frac{e^2}{4\pi\hbar}\left(1-\frac{h_0}{\sqrt{h_0^2+\frac{2\gamma_0^2m\epsilon_F}{\hbar^2}+\left(\frac{\gamma_0^2m}{\hbar}\right)^2}}\right)\Theta(h_0-\epsilon_F). \quad (20)$$

One can find that the sign of $\sigma_{yx}^{\mathrm{II}}$ in the case of $k$-linear Dresselhaus SOI is opposite to that of $k$-linear Rashba SOI given by Nunner *et al.*[19] However, when $k^3$ term of Dresselhaus SOI in Eq. (2) is also considered, it is hard to give an analytic result and so we will numerically calculate $\sigma_{yx}^{\mathrm{II}}$ of Eq. (19) in what follows.

To calculate $\sigma_{yx}^{\mathrm{I}}$, we take vertex corrections which can be of similar magnitude as the bare bubble and thus we divide $\sigma_{yx}^{\mathrm{I}}$ into two parts,[19] $\sigma_{yx}^{\mathrm{I}}=\sigma_{yx}^{\mathrm{I},b}+\sigma_{yx}^{\mathrm{I},l}$, where $\sigma_{yx}^{\mathrm{I},b}$ is the bare bubble contribution and $\sigma_{yx}^{\mathrm{I},l}$ is the ladder vertex corrections. First, we can give the general form of $\sigma_{yx}^{\mathrm{I},b}$ for Eq. (4)

$$\sigma_{yx}^{\mathrm{I},b} = \frac{e^2}{2\pi\hbar}\int\int\frac{kdkd\theta}{(2\pi)^2}\mathrm{Tr}[v_yG^R(\epsilon_F)v_xG^A(\epsilon_F)]$$
$$= \frac{e^2}{8\pi^2\hbar}\int d\theta\left[\frac{\nu_+(\Omega_1^++\Omega_2^+)}{\Gamma_+}+\frac{\nu_-(\Omega_1^--\Omega_2^-)}{\Gamma_-} - \left(\frac{\nu_+\Omega_3^+}{\lambda_+}+\frac{\nu_-\Omega_3^-}{\lambda_-}\right)\right], \quad (21)$$

where $\Omega_i^{\pm}\equiv\Omega_i(k_\pm,\theta)$ $(i=1,2,3)$ with $\Omega_i(k,\theta)$ defined by

$$\Omega_1 = \frac{\hbar}{m}\sum_\alpha \mathrm{Im}(k_yd_{\alpha x}g_{\alpha+1}g_{\alpha+2}^*-k_xd_{\alpha y}g_{\alpha+1}g_{\alpha+2}^*) + \sum_\alpha(d_{\alpha y}d_{\alpha+1,x}+d_{\alpha x}d_{\alpha+1,y})\mathrm{Re}(g_\alpha g_{\alpha+1}^*) + \frac{1}{2}\sum_\alpha d_{\alpha x}d_{\alpha y}\Pi_\alpha+\frac{\hbar^2k_xk_y}{2m^2}\Pi_0, \quad (22)$$

$$\Omega_2 = \frac{\hbar k_x}{m}\sum_\alpha d_{\alpha y}\,\mathrm{Re}(g_\alpha)+\frac{\hbar k_y}{m}\sum_\alpha d_{\alpha x}\,\mathrm{Re}(g_\alpha) - \sum_\alpha(d_{\alpha x}d_{\alpha+1,y}-d_{\alpha y}d_{\alpha+1,x})\mathrm{Im}(g_{\alpha+2}), \quad (23)$$

$$\Omega_3 = -\frac{\hbar k_x}{m}\sum_\alpha d_{\alpha y}\,\mathrm{Im}(g_\alpha)-\frac{\hbar k_y}{m}\sum_\alpha d_{\alpha x}\,\mathrm{Im}(g_\alpha) - \sum_\alpha(d_{\alpha x}d_{\alpha+1,y}-d_{\alpha y}d_{\alpha+1,x})\mathrm{Re}(g_{\alpha+2}), \quad (24)$$

where $d_{\alpha\beta}=\partial d_\alpha/(\hbar\partial k_\beta)$ $(\alpha,\beta=x,y,z)$, $\Pi_0=1+|g_x|^2+|g_y|^2+|g_z|^2$, $\Pi_x=1+|g_x|^2-|g_y|^2-|g_z|^2$, $\Pi_y=1-|g_x|^2+|g_y|^2-|g_z|^2$, $\Pi_z=1-|g_x|^2-|g_y|^2+|g_z|^2$, and $\mathrm{Im}(\cdots)$ and $\mathrm{Re}(\cdots)$ denote to take the imaginary part and the real part, respectively. In Eqs. (22)–(24) and the following texts, the definitions $x+1=y$, $y+1=z$, and $z+1=x$ are used. For Eq. (3), we have $d_{zx}=d_{zy}=0$, $d_{xx}=(-\gamma_0+\gamma k_y^2)/\hbar$, $d_{xy}=2\gamma k_xk_y/\hbar$, $d_{yx}=-2\gamma k_yk_x/\hbar$, and $d_{yy}=(\gamma_0-\gamma k_x^2)/\hbar$.

Then, we calculate the ladder vertex corrections $\sigma_{yx}^{\mathrm{I},l}$. In the ladder approximation, the vertex correction of electric velocity $\mathcal{T}_x$ satisfies the self-consistent vertex equation[38]

$$\mathcal{T}_x = v_x+n_iV_0^2\int\frac{d^2k}{(2\pi)^2}G^R(\epsilon_F)\mathcal{T}_xG^A(\epsilon_F). \quad (25)$$

If the second term of right-hand side in Eq. (25) is defined as $\Upsilon_x$, then one can get

$$\Upsilon_x = n_iV_0^2\int\frac{d^2k}{(2\pi)^2}G^R(\epsilon_F)v_xG^A(\epsilon_F) + n_iV_0^2\int\frac{d^2k}{(2\pi)^2}G^R(\epsilon_F)\Upsilon_xG^A(\epsilon_F)\equiv\tilde{v}_x+\tilde{\Upsilon}_x. \quad (26)$$

The first term $\tilde{v}_x$ of right-hand side in Eq. (26) is given

$$\tilde{v}_x = n_iV_0^2\int\frac{d^2k}{(2\pi)^2}G^R(\epsilon_F)v_xG^A(\epsilon_F)=\sum_{\mu=0,x,y,z}c_\mu\sigma_\mu, \quad (27)$$

where $\sigma_0$ is the two by two identity matrix and

$$c_\mu = \frac{n_iV_0^2}{4\pi}\int d\theta\left[\frac{\nu_+(\omega_{\mu1}^++\omega_{\mu2}^+)}{\Gamma_+}+\frac{\nu_-(\omega_{\mu1}^--\omega_{\mu2}^-)}{\Gamma_-} - \left(\frac{\nu_+\omega_{\mu3}^+}{\lambda_+}+\frac{\nu_-\omega_{\mu3}^-}{\lambda_-}\right)\right], \quad (28)$$

where $\omega_{\mu i}^{\pm}\equiv\omega_{\mu i}(k_\pm,\theta)$ $(i=1,2,3)$ with $\omega_{\mu i}(k,\theta)$ defined by

$$\omega_{01} = -\frac{1}{2}\sum_\alpha d_{\alpha x}\,\mathrm{Im}(g_{\alpha+1}^*g_{\alpha+2})+\frac{\hbar k_x}{4m}\Pi_0,$$

$$\omega_{02} = \frac{1}{2}\sum_\alpha d_{\alpha x}\,\mathrm{Re}(g_\alpha), \quad \omega_{03} = -\frac{1}{2}\sum_\alpha d_{\alpha x}\,\mathrm{Im}(g_\alpha),$$

$$\omega_{\alpha 1} = \frac{d_{\alpha x}}{4}\Pi_\alpha+\frac{1}{2}\mathrm{Re}(d_{\alpha+1,x}g_\alpha g_{\alpha+1}^*+d_{\alpha+2,x}g_\alpha g_{\alpha+2}^*) - \frac{\hbar k_x}{2m}\mathrm{Im}(g_{\alpha+1}g_{\alpha+2}^*),$$

$$\omega_{\alpha 2}=\frac{1}{2}\mathrm{Im}(d_{\alpha+1,x}g_{\alpha+2}-d_{\alpha+2,x}g_{\alpha+1})+\frac{\hbar k_x}{2m}\mathrm{Re}(g_\alpha),$$

$$\omega_{\alpha 3}=\frac{1}{2}\mathrm{Re}(d_{\alpha+1,x}g_{\alpha+2}-d_{\alpha+2,x}g_{\alpha+1})-\frac{\hbar k_x}{2m}\mathrm{Im}(g_\alpha).$$

The renormalized vertex $\Upsilon_x$ has the general solution

$$\Upsilon_x=\sum_{\mu=0,x,y,z} b_\mu\sigma_\mu. \qquad (29)$$

Substituting Eqs. (27) and (29) into Eq. (26), one can obtain

$$\Upsilon_x=\sum_\mu\left[c_\mu\sigma_\mu+n_iV_0^2\int\frac{d^2k}{(2\pi)^2}G^R(\epsilon_F)b_\mu\sigma_\mu G^A(\epsilon_F)\right]=\sum_\mu\left[c_\mu\sigma_\mu+b_\mu\sum_\nu\chi^\nu_\mu\sigma_\nu\right], \qquad (30)$$

where

$$\chi^\nu_\mu=\frac{n_iV_0^2}{4\pi}\int d\theta\left[\frac{\nu_+(\Theta^{\nu+}_{\mu1}+\Theta^{\nu+}_{\mu2})}{\Gamma_+}+\frac{\nu_-(\Theta^{\nu-}_{\mu1}-\Theta^{\nu-}_{\mu2})}{\Gamma_-}-\left(\frac{\nu_+\Theta^{\nu+}_{\mu3}}{\lambda_+}+\frac{\nu_-\Theta^{\nu-}_{\mu3}}{\lambda_-}\right)\right], \qquad (31)$$

where $\Theta^{\nu\pm}_{\mu i}\equiv\Theta^\nu_{\mu i}(k_\pm,\theta)$ $(i=1,2,3$ and $\mu,\nu=0,x,y,z)$ with $\Theta^\nu_{\mu i}(k,\theta)$ defined by $\Theta^\mu_{\mu1}=\frac{\Pi_\mu}{4}$, $\Theta^\mu_{\mu2}=\Theta^\mu_{\mu3}=0$, $\Theta^\alpha_{01}=-\Theta^0_{\alpha1}=-\frac{1}{2}\mathrm{Im}(g_{\alpha+1}g^*_{\alpha+2})$, $\Theta^\alpha_{02}=\Theta^0_{\alpha2}=\frac{1}{2}\mathrm{Re}(g_\alpha)$, $\Theta^\alpha_{03}=\Theta^0_{\alpha3}=-\frac{1}{2}\mathrm{Im}(g_\alpha)$, $\Theta^{\alpha+1}_{\alpha1}=\Theta^\alpha_{\alpha+1,1}=\frac{1}{2}\mathrm{Re}(g_\alpha g^*_{\alpha+1})$, $\Theta^{\alpha+1}_{\alpha2}=-\Theta^\alpha_{\alpha+1,2}=-\frac{1}{2}\mathrm{Im}(g_{\alpha+2})$, and $\Theta^{\alpha+1}_{\alpha3}=-\Theta^\alpha_{\alpha+1,3}=-\frac{1}{2}\mathrm{Re}(g_{\alpha+2})$. Thus the coefficients $b_\mu$ in Eq. (29) are given by the following equations:

$$b_\mu=\sum_\nu Q_{\mu\nu}c_\nu, \qquad (32)$$

where $Q_{\mu\nu}$ are the elements of the four by four matrix

$$Q=\begin{pmatrix}(1-\chi^0_0) & -\chi^0_x & -\chi^0_y & -\chi^0_z\\ -\chi^x_0 & (1-\chi^x_x) & -\chi^x_y & -\chi^x_z\\ -\chi^y_0 & -\chi^y_x & (1-\chi^y_y) & -\chi^y_z\\ -\chi^z_0 & -\chi^z_x & -\chi^z_y & (1-\chi^z_z)\end{pmatrix}^{-1}. \qquad (33)$$

The ladder diagrams are therefore given by

$$\sigma^{\mathrm{I},l}_{yx}=\frac{e^2}{2\pi\hbar}\sum_\mu\int\frac{d^2k}{(2\pi)^2}\mathrm{Tr}[v_yG^R(\epsilon_F)b_\mu\sigma_\mu G^A(\epsilon_F)]=\frac{e^2}{8\pi^2\hbar}\int d\theta\left[\frac{\nu_+(\tilde{\Omega}^+_1+\tilde{\Omega}^+_2)}{\Gamma_+}+\frac{\nu_-(\tilde{\Omega}^-_1-\tilde{\Omega}^-_2)}{\Gamma_-}-\left(\frac{\nu_+\tilde{\Omega}^+_3}{\lambda_+}+\frac{\nu_-\tilde{\Omega}^-_3}{\lambda_-}\right)\right], \qquad (34)$$

where $\tilde{\Omega}^\pm_i\equiv\tilde{\Omega}_i(k_\pm,\theta)$ $(i=1,2,3)$ and $\tilde{\Omega}_i(k,\theta)$ can be obtained by replacing $\hbar k_x/m$ with $b_0$ and $d_{\alpha x}$ with $b_\alpha$ $(\alpha=x,y,z)$ in Eqs. (22)–(24) of $\Omega_i(k,\theta)$.

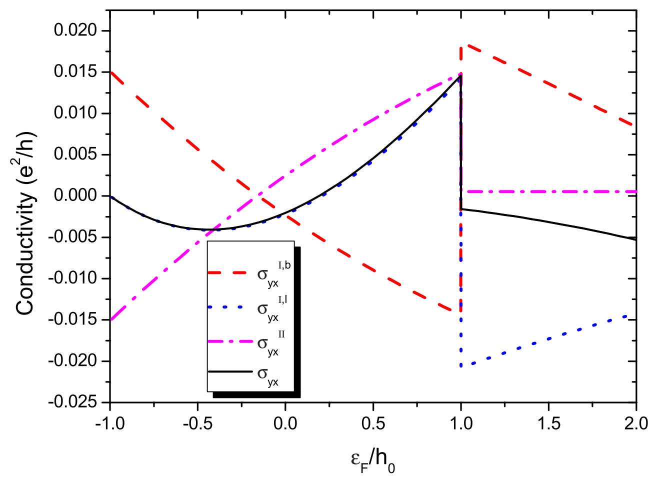


FIG. 1. (Color online) Anomalous Hall conductivity plotted as a function $\epsilon_F/h_0$. Numerical calculation is performed in the weak scattering limit for a GaSb QW with $m=0.041m_0$, $\gamma=187$ eV Å$^3$, $a=5$ nm, and $h_0=100$ meV. The solid (black) line corresponds to the total intrinsic anomalous Hall conductivity $\sigma_{yx}$, the dashed (red) line to $\sigma^{\mathrm{I},b}_{yx}$, the dotted (blue) line to $\sigma^{\mathrm{I},l}_{yx}$, and the dashed-dotted (magenta) line to $\sigma^{\mathrm{II}}_{yx}$.

## IV. NUMERICAL RESULTS AND DISCUSSIONS

We start our discussion of the numerical results in the weak scattering limit. We consider a GaSb QW with an effective mass of conduction electron $m=0.041m_0$ (here $m_0$ is the free electron mass), the spin-orbit coefficient $\gamma=187$ eV Å$^3$.[31] The value of $\gamma_0$ is dependent on the thickness of QW, here we assume the thickness $a=5$ nm and so $\gamma_0=73.82$ eV Å. Numerical integrations give that $\Gamma_x=\Gamma_y=0$ and $\Gamma_z\neq0$ whether $\epsilon_F>h_0$ or $-h_0<\epsilon_F<h_0$. In the previous case for Rashba type[19] or pure $k$-linear Dresselhaus-type SOI, analytic results give $\Gamma_z=0$ for the situation where both subbands are partially occupied (i.e., $\epsilon_F>h_0$) and $\Gamma_z\neq0$ when only the majority band is partially occupied (i.e.,

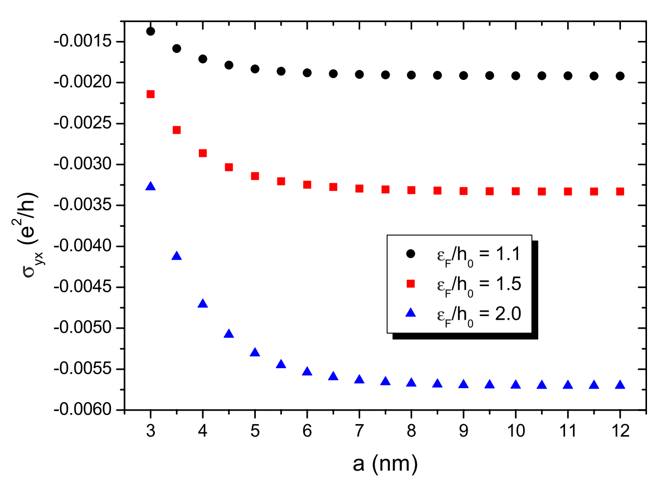


FIG. 2. (Color online) The total intrinsic anomalous Hall conductivity $\sigma_{yx}$ for different Fermi energies plotted as a function of the thickness of QW in the situation that both subbands are partially occupied. The (black) circles correspond to $\epsilon_F/h_0=1.1$, the (red) squares to $\epsilon_F/h_0=1.5$, and the (blue) triangles to $\epsilon_F/h_0=2.0$.

$-h_0 < \epsilon_F < h_0$). In Fig. 1, the total intrinsic anomalous Hall conductivity $\sigma_{yx}$ and the three contributions to the conductivity (i.e., $\sigma_{yx}^{\mathrm{I},b}$, $\sigma_{yx}^{\mathrm{I},l}$, and $\sigma_{yx}^{\mathrm{II}}$) are plotted as a function $\epsilon_F/h_0$. When only the majority band is partially occupied (i.e., $-h_0 < \epsilon_F < h_0$), one can observe the sign change in anomalous Hall conductivity $\sigma_{yx}$. Near $\epsilon_F/h_0=1$, there is a sharp change in $\sigma_{yx}$. When both subbands are occupied (i.e., $\epsilon_F > h_0$), $\sigma_{yx}$ is nonvanishing and its absolute value increases with the increment of the Fermi energy while $\sigma_{yx}$ is zero for the case of Rashba SOI.[19] It is easy to check that if the $k^3$ term of Dresselhaus SOI is neglected and only $k$-linear term of Dresselhaus SOI is kept, analogous to Rashba SOI, the intrinsic AHE vanishes for the case of $\epsilon_F > h_0$. The total intrinsic anomalous Hall conductivity $\sigma_{yx}$ for different Fermi energies ($\epsilon_F > h_0$) is plotted as a function of the thickness of QW (Fig. 2). We find that the absolute value of the intrinsic anomalous Hall conductivity $\sigma_{yx}$ increases with the increment of the thickness $a$ when both subbands are partially occupied. Note that with increasing of the thickness $a$, the coefficient $\gamma_0$ of the $k$-linear term of Dresselhaus SOI decreases. In fact the $k^3$ term of Dresselhaus SOI plays a key role in this nonzero conductivity, which is analogous to the intrinsic SHE.

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